Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at http://about.jstor.org/participate-jstor/individuals/early-journal-content.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

CALCULUS.

230. Proposed by C. N. SCHMALL, College of the City of New York.

The greatest rectangle is inscribed in an ellipse, and the greatest ellipse in that rectangle, again the greatest rectangle in that (second) ellipse, and the greatest ellipse in that (second) rectangle, and so on *ad infinitum;* show that the sum of all the inscribed rectangles is equal to the area of the rectangle circumscribed about the given ellipse.

Solution by J. E. SANDERS, Reinersville, Ohio, and A. H. HOLMES, Brunswick, Maine.

Let $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$, be the equation of the ellipse. Let $2x_1$ be the length of its maximum inscribed rectangle. Then $2y_1=2\sqrt{(a^2b^2-b^2x_1^2)}=$its width, and the area of the rectangle is $4x_1y_1=4bx_1\sqrt{(a^2-x_1^2)}/a=$maximum.

$\therefore 4b\sqrt{(a_2-x_1^2)}\,\dfrac{4bx^2}{\sqrt{(a^2-x_1^2)}}=0$; from which $x_1=\frac{1}{2}\sqrt{2}\,a=a_1$. Hence, $y_1=\frac{1}{2}\sqrt{2}\,b=b_1$. Hence its area$=4a_1b_1=2ab$. The equation of the ellipse inscribed in this rectangle is $\dfrac{x^2}{a_1^2}+\dfrac{y^2}{b_1^2}=1$. Hence, the length of the maximum rectangle inscribed in this ellipse is $2(\frac{1}{2}\sqrt{2}\,a_1)=2a_2$, its width is $2(\frac{1}{2}\sqrt{2}\,b_1)=2b_2$, and its area is $4a_2b_2=2a_1b_1=ab$. By induction, the area of the nth rectangle$=\frac{1}{2}$ of the area of the $(n-1)$th rectangle, or $a_nb_n=\frac{1}{2}a_{n-1}b_{n-1}$. Hence, the sum of all the rectangles is $S=2ab+ab+\frac{1}{2}ab+\frac{1}{4}ab+\frac{1}{8}ab+\ldots$ *ad infinitum*$=4ab$, which is the area of the rectangle circumscribing the original ellipse.

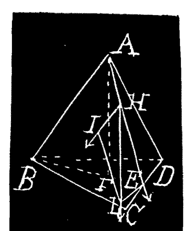
Also solved by G. B. M. Zerr, J. Scheffer, and G. W. Greenwood.

MECHANICS.

193. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

Three light smoothly jointed rods stand like a tripod—the three edges of a regular tetrahedron. A rectangular board, weight w, stands on this like an easel. Find the thrust on the rod which does not touch the easel.

Solution by G. B. M. ZERR, Ph. D., Parsons, W. Va.



Let $AB=a$ be one of the equal rods jointed at A; H the center of gravity of the board, weight w; HL the direction of the weight w. Resolve HL into the components HE along the median AE, and HI parallel to AB. Then HI is the thrust required. Now $AE=BE=\frac{1}{2}a\sqrt{3}$. Draw AF perpendicular to BE. Then $BF=\frac{2}{3}BE=\frac{1}{3}a\sqrt{3}$. $EF=\frac{1}{6}a\sqrt{3}$.

$\cos BAE=\cos IHE=\frac{1}{3}\sqrt{3}$, $\sin IHE=\frac{1}{3}\sqrt{6}$.